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PARAMETRIC TENSION BETWEEN EVEN AND ODD MULTIPOLE DATA OF WMAP POWER SPECTRUM: UNACCOUNTED CONTAMINATION OR MISSING PARAMETERS?

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ABSTRACT

There exist power contrast in even and odd multipoles of WMAP power spectrum at low and intermediate multipole range. This anomaly is explicitly associated with the angular power spectrum, which are heavily used for cosmological model fitting. Having noted this, we have investigated whether even(odd) multipole data set is individually consistent with the WMAP concordance model. Our investigation shows the WMAP concordance model does not make a good fit for even(odd) multipole data set, which indicates parametric tension between even and odd multipole data set. Noting tension is highest in primordial power spectrum parameters, we have additionally considered a running spectral index, but find tension increases to even a higher level. We believe these parametric tensions may be indications of unaccounted contamination or imperfection of the model.

Subject headings: cosmic microwave background radiation — methods: data analysis

1. INTRODUCTION

For the past years, there have been great successes in measurement of CMB anisotropy by ground and satellite observations (Runyan and et al. 2003; Ade and et al. 2008; Nolta et al. 2009; Hinderks et al. 2009; Hinshaw and et al. 2009; Dunkley and et al. 2009; Pryke and et al. 2009; Reichardt and et al. 2009; Larson et al. 2010; Jarosik et al. 2010). By comparing the angular power spectrum of the CMB anisotropy with theoretical predictions, we may impose strong constraints on cosmological models (Liddle and Lyth 2000; Dodelson 2003; Mukhanov 2005; Weinberg 2008). In spite of remarkable goodness of fit (Komatsu and et al. 2009, 2010), there are some features of WMAP data, which are not well explained by the WMAP concordance model (Chiang et al. 2003; de Oliveira-Costa et al. 2004; Copi et al. 2004; Eriksen et al. 2004a; Cruz et al. 2005; Land and Magueijo 2005; Kim and Naselsky 2009a; Copi et al. 2009; Kim and Naselsky 2009b, 2010a,b; Bennett et al. 2010; Copi et al. 2010). In particular, the power contrast anomaly between even and odd multipoles is explicitly associated with the angular power spectrum, which are mainly used to fit cosmological models (Land and Magueijo 2005; Kim and Naselsky 2010a,b; Gruppuso et al. 2010; Bennett et al. 2010). Having noted this, we have investigated whether even(odd) multipole data set is consistent with the WMAP concordance model. Our investigation shows there exist some level of tension, which may be an indication of unaccounted contamination or missing ingredients in the assumed parametric model such as the flat ΛCDM model.

2. EVEN(ODD) MULTIPOLE DATA AND COSMOLOGICAL MODEL FITTING

We may consider CMB anisotropy as the sum of even and odd parity functions:

$$T(\hat{\mathbf{n}}) = T^{+}(\hat{\mathbf{n}}) + T^{-}(\hat{\mathbf{n}}), \tag{1}$$

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where

$$T^{+}(\hat{\mathbf{n}}) = \frac{T(\hat{\mathbf{n}}) + T(-\hat{\mathbf{n}})}{2}, \tag{2}$$

$$T^{-}(\hat{\mathbf{n}}) = \frac{T(\hat{\mathbf{n}}) - T(-\hat{\mathbf{n}})}{2}. \tag{3}$$

Using the parity property of spherical harmonics $Y_{lm}(\hat{\mathbf{n}}) = (-1)^l\, Y_{lm}(-\hat{\mathbf{n}})$ (Arfken and Weber 2000), it is straightforward to show

$$T^{+}(\hat{\mathbf{n}}) = \sum_{l=\mathrm{even}} \sum_{m} a_{lm}\, Y_{lm}(\hat{\mathbf{n}}), \tag{4}$$

$$T^{-}(\hat{\mathbf{n}}) = \sum_{l=\mathrm{odd}} \sum_{m} a_{lm}\, Y_{lm}(\hat{\mathbf{n}}). \tag{5}$$

Obviously, the power spectrum of even and odd multipoles are associated with $T^{+}(\hat{\mathbf{n}})$ and $T^{-}(\hat{\mathbf{n}})$ respectively. Given the ΛCDM model, we do not expect any features distinct between even and odd multipoles. However, there have been reported power contrast between even and odd multipoles of WMAP TT power spectrum (Land and Magueijo 2005; Kim and Naselsky 2010a,b; Gruppuso et al. 2010; Bennett et al. 2010). At lowest multipoles ($2 \leq l \leq 22$), there is odd multipole preference (i.e. power excess in odd multipoles and deficit in even multipoles) (Land and Magueijo 2005; Kim and Naselsky 2010a,b; Gruppuso et al. 2010), and even multipole preference at intermediate multipoles ($200 \leq l \leq 400$) (Bennett et al. 2010). Additionally, we have investigated TE correlation, and noticed odd multipole preference at ($100 \lesssim l \lesssim 200$) and even multipole preference at ($200 \lesssim l \lesssim 400$), though its statistical significance is not high enough, due to low Signal-to-Noise Ratio of polarization data. Not surprisingly, these power contrast anomalies are explicitly associated with the angular power spectrum data, which are mainly used to fit cosmological models. Having noted this, we have investigated whether the even(odd) multipole data set is consistent with the concordance model. For a cosmological model, we have considered ΛCDM + SZ effect + weak-lensing, where cosmological parameters are

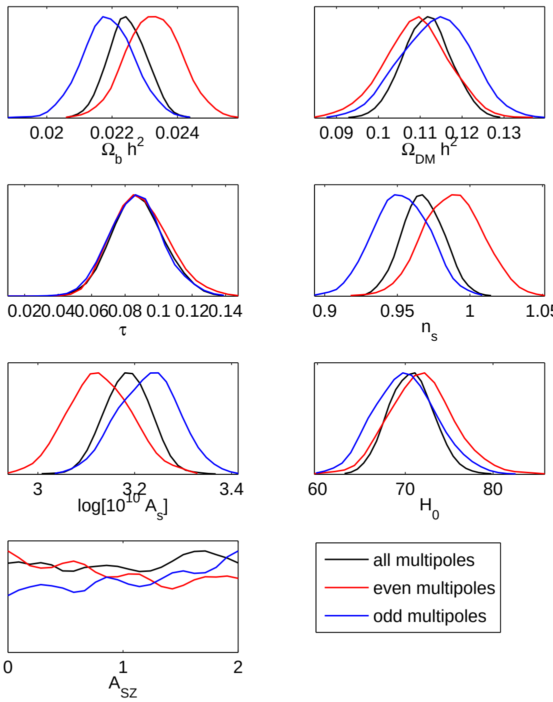

Figure 1. Marginalized likelihood of cosmological parameters (ΛCDM + sz + lens), given whole or even(odd) multipole data.

$\lambda \in \{\Omega_b, \Omega_c, \tau, n_s, A_s, A_{sz}, H_0\}$. For data constraints, we have used the WMAP 7 year TT and TE power spectrum data, which have been estimated from the ILC map, and cut-sky V and W band maps (Larson et al. 2010). It should be noted that we have used the WMAP power spectrum via the WMAP team's likelihood code.

In order to use only even(odd) multipole data, we have made slight modifications to the WMAP team's likelihood code, where the Blackwell-Rao estimator and the MASTER pseudo-Cl estimator are modified respectively for low multipoles ($l \leq 32$) and high multipoles ($l > 32$) (Eriksen et al. 2004b; Wandelt et al. 2001; Hivon et al. 2002; Larson et al. 2010). Note that we have also rederived the Fisher matrix in accordance with even(odd) multipole data subset. Additionally, the beam and point source corrections are modified accordingly. Hereafter, we shall denote WMAP CMB data of whole, even and odd multipoles by D_0, D_2 and D_3 respectively. Note that even/odd multipole splitting are made for TT and TE power spectrum up to the multipoles of WMAP sensitivity (i.e. $l \leq 1200$ for TT and $l \leq 800$ for TE). Using `CosmoMC` with the modified WMAP likelihood code, we have explored the parameter space on a MPI cluster with 6 chains (Lewis and Bridle 2002; Eriksen et al. 2004b; Lewis and Bridle 2006; Larson et al. 2010). For the convergence criterion, we have adopted the Gelman and Rubin's "variance of chain means" and set the R-1 statistic to 0.03 for stopping criterion (Gelman and Rubin 1992; Brooks and Gelman 1998).

In Fig. 1, we show the marginalized likelihood of parameters, which are obtained from the run of a `CosmoMC` with D_0, D_2 and D_3 respectively. In Table 1, we show the

Table 1
cosmological parameters (ΛCDM + sz + lens)

	λ_0	λ_2	λ_3
$\Omega_b\, h^2$	0.0226 ± 0.0006	0.0231 ± 0.0008	0.0217 ± 0.0008
$\Omega_c\, h^2$	0.112 ± 0.006	0.109 ± 0.008	0.115 ± 0.008
τ	0.0837 ± 0.0147	0.0913 ± 0.0157	0.0859 ± 0.015
n_s	0.964 ± 0.014	0.989 ± 0.02	0.949 ± 0.019
$\log[10^{10} A_s]$	3.185 ± 0.047	3.132 ± 0.065	3.239 ± 0.062
H_0	70.53 ± 2.48	71.73 ± 3.59	69.68 ± 3.47
$A_{\rm sz}$	$1.891^{+0.109}_{-1.891}$	$0.169^{+1.831}_{-0.169}$	$0.89^{+1.11}_{-0.89}$

best-fit parameters and 1 σ confidence intervals, where λ_2 and λ_3 denote the best-fit values of D_2 and D_3 respectively. The parameter set λ_0 are the best-fit values of whole data D_0, and accordingly corresponds to the WMAP concordance model. As shown in Fig. 1 and Table 1, we find non-negligible tension especially in parameters of primordial power spectrum. It is worth to note that the best-fit spectral index of even multipole data (i.e. D_2) is close to a flat spectrum (i.e. $n_s = 1$), while the result from the whole data rule out the flat spectrum by more than 2σ.

There is a likelihood-ratio test, which allows us to determine the rejection region of an alternative hypothesis, given a null hypothesis (Mood 1974; Cox and Hinkley 1979; Lupton 1993; K. F. Riley M. P. Hobson 2006). By setting sets of parameters to a null hypothesis and an alternative hypothesis, we may investigate whether two sets of parameters are consistent with each other. Therefore, we have evaluated the following in order to assess parametric tension:

$$\frac{\mathcal{L}(\lambda_j|D_i)}{\mathcal{L}(\lambda_i|D_i)},$$

where parameter set λ_i and λ_j correspond to a null hypothesis and an alternative hypothesis respectively. In

Table 2
the likelihood ratio: ΛCDM + sz + lens

	$\mathcal{L}(\lambda_0|D_0)$	$\mathcal{L}(\lambda_2|D_0)$	$\mathcal{L}(\lambda_3|D_0)$
$\mathcal{L}(\lambda_0|D_0)$	1	0.076	0.0099
	$\mathcal{L}(\lambda_0|D_2)$	$\mathcal{L}(\lambda_2|D_2)$	$\mathcal{L}(\lambda_3|D_2)$
$\mathcal{L}(\lambda_2|D_2)$	0.16	1	2×10^{-4}
	$\mathcal{L}(\lambda_0|D_3)$	$\mathcal{L}(\lambda_2|D_3)$	$\mathcal{L}(\lambda_3|D_3)$
$\mathcal{L}(\lambda_3|D_3)$	0.16	0.0022	1

Table 2, we show the likelihood ratio, where the quantities used for the numerator and denominator are indicated in the uppermost row and leftmost column. As shown by $\mathcal{L}(\lambda_0|D_2)/\mathcal{L}(\lambda_2|D_2)$ and $\mathcal{L}(\lambda_0|D_3)/\mathcal{L}(\lambda_3|D_3)$, the WMAP concordance model (i.e. λ_0) does not make a good fit for even(odd) multipole data set. Besides, there exist significant tension between two data subsets, as indicated by very small values of $\mathcal{L}(\lambda_3|D_2)/\mathcal{L}(\lambda_2|D_2)$ and $\mathcal{L}(\lambda_2|D_3)/\mathcal{L}(\lambda_3|D_3)$. The parameter likelihood, except for $A_{\rm sz}$, follows the shape of Gaussian functions, as shown in Fig. 1. For a likelihood of Gaussian shape, the likelihood ratio 0.1353 and 0.0111 correspond to 2σ and 3σ significance level respectively. From Table 2, we may see most of the ratio indicates $\sim 2\sigma$ tension or even higher.

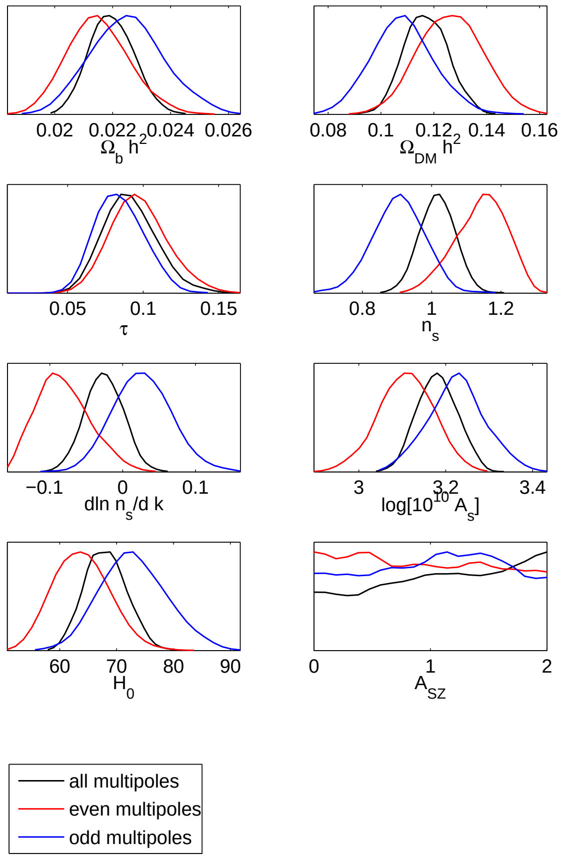

Figure 2. Marginalized likelihood of cosmological parameters (ΛCDM + sz + lens + run), given whole, even and odd multipole data respectively.

Table 3
the likelihood ratio: ΛCDM + sz + lens + run

	$\mathcal{L}(\lambda_0\|D_0)$	$\mathcal{L}(\lambda_2\|D_0)$	$\mathcal{L}(\lambda_3\|D_0)$
$\mathcal{L}(\lambda_0\|D_0)$	1	3.5×10^{-4}	0.0078
	$\mathcal{L}(\lambda_0\|D_2)$	$\mathcal{L}(\lambda_2\|D_2)$	$\mathcal{L}(\lambda_3\|D_2)$
$\mathcal{L}(\lambda_2\|D_2)$	0.06	1	2.3×10^{-5}
	$\mathcal{L}(\lambda_0\|D_3)$	$\mathcal{L}(\lambda_2\|D_3)$	$\mathcal{L}(\lambda_3\|D_3)$
$\mathcal{L}(\lambda_3\|D_3)$	0.042	5.8×10^{-7}	1

As discussed previously, the tension is highest in parameters of primordial power spectrum, which may be an indication of missing parameters in primordial power spectrum (e.g. a running spectral index). Therefore, we have additionally considered a running spectral index $dn_s/d\ln k$ and repeated our investigation. Surprisingly, we find tension increases to even a higher level. We show the marginalized parameter likelihoods and the likelihood ratios in Fig. 2 and Table 3, where we find tension is also highest in the primordial power spectrum parameters.

3. DISCUSSION

The WMAP power contrast anomaly between even and odd multipoles is explicitly associated with the angular power spectrum data, which are mainly used to fit a cosmological model. Having noted this, we have investigated whether even(odd) low multipole data set is consistent with the WMAP concordance model. Our investigation shows there exists some level of tension. Noting tension is highest in primordial power spectrum parameters, we have additionally considered the running of a spectral index $dn_s/d\ln k$, but find tension increases to even a higher level. These parametric tensions may be indications of unaccounted contamination or missing ingredients of the assumed parametric model (i.e. the flat ΛCDM with/without a running spectral index). Therefore, we believe these parametric tension deserve further investigation. The Planck surveyor data, which possesses wide frequency coverage and systematics distinct from the WMAP, may allow us to resolve this tension.

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